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Spin relaxation of two-dimensional electrons with a hierarchy of spin–orbit couplings

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Abstract

The density-matrix formalism is applied to calculate the spin-relaxation time for two-dimensional systems with a hierarchy of spin–orbit couplings, such as Rashba-type, Dresselhaus-type and strain-induced. It is found that the spin-relaxation time can be infinite if those coupling strengths α, β, γ_1 and γ_2 satisfy either condition (i) $\alpha = \beta$, $\gamma_1 = 0$ or (ii) $\alpha = -\beta$, $\gamma_2 = 0$, which correspond to the vanishing Yang–Mills fields. The effect caused by the application of an external magnetic field is also discussed. It is found that the longitudinal spin component can possess infinite life time when the spin components, the Larmor precession frequency and the external magnetic field satisfy certain relations for which the Yang–Mills fields become zero.

1. Introduction

Spintronics [1], or spin-based electronics [2], has been given increasing attention during the last decade. One important issue in this research area is the manipulation of spin-polarized electrons with the help of an electric field [3–7]. A system with spin–orbit couplings makes these efforts possible and thus brings great interests from both academic and practical aspects [3–6]. However, the problem of the loss of the average microscopic spin is crucial in experimental data analysis and applicable device construction. The study of the spin-relaxation mechanisms of two-dimensional electrons is thus very important.

Spin relaxation exhibits some properties of the spin dynamics, which plays an inevitable role in realizing applicable spintronics devices. The main mechanism of spin relaxation in systems lacking inversion symmetry is the D'yakonov–Perel mechanism [8, 9], in which the spin of the electron precesses due to an effective **k**-dependent magnetic field. For electrons in two-dimensional semiconductor heterostructures or quantum wells, the structure inversion asymmetry brings about the Rashba spin–orbit coupling [10–12], while the bulk inversion asymmetry in the A_3B_5 compounds leads to the Dresselhaus [13] spin–orbit coupling. The spin-relaxation time in some semiconductors with both Rashba and Dresselhaus couplings was calculated by analysing the condition of spin decay [14, 15], and the effect of external magnetic fields was discussed [16] furthermore. An infinite spin-relaxation time [17] was predicted in a system with equal Rashba and Dresselhaus coupling constants by making use of an $SU(2)$ symmetry in k-space. It is important to understand the spin-relaxation mechanism and the condition for infinite spin-relaxation time to occur, which would be helpful for overcoming the difficulties in spin-based information processes.

In this paper, we develop the aforementioned theory of spin relaxation to describe two-dimensional electron systems in the presence of a $U(1)$ Maxwell field and $SU(2)$ Yang–Mills fields. Such a system can be realized in certain semiconductor materials where the spin–orbit couplings, such as Rashba-type, Dresselhaus-type and strain-induced, play crucial roles. Using the density-matrix formalism, we calculate the spin-relaxation time for a system with a hierarchy of spin–orbit couplings. In the absence of the Maxwell magnetic field, infinity of the spin-relaxation time occurs if the spin–orbit couplings α, β, γ_1 and γ_2 satisfy the condition in which the Yang–Mills fields vanish. In order to capture the physical essence of the emergence of an infinite spin-relaxation time, we further study the effect of the external magnetic field on the same systems and find that the longitudinal spin component can also possess infinite life times when the spin orientation, the Larmor precession frequency and external magnetic field satisfy some relations. Based on the analysis of spin–orbit systems with or without Maxwell magnetic fields, we expose a physical picture for a clear understanding of the infinite spin-relaxation time, which is helpful for the design of spin-based devices.

2. Spin relaxation arising from spin–orbit couplings

To start with a general formalism, we consider the Schrödinger equation for a particle moving in an external $U(1)$ Maxwell field and $SU(2)$ Yang–Mills fields [18],

$$\begin{aligned} &\mathrm{i}\hbar\frac{\partial}{\partial t}\Psi(r,t) = H\Psi(r,t), \\ &H = \frac{1}{2m}\left(\hat{\mathbf{p}} - \frac{e}{c}\mathbf{A} - \eta\mathcal{A}^a\hat{\tau}^a\right)^2 + eA_0 + \eta\mathcal{A}_0^a\hat{\tau}^a, \end{aligned} \tag{1}$$

where Ψ is a two-component wavefunction, $\mathbf{A}_\mu = (A_0, A_i)$ denotes the vector potential of the Maxwell electromagnetic field, and $\mathbb{A}_\mu = \mathcal{A}_\mu^a\hat{\tau}^a$ that of Yang–Mills field, with $\hat{\tau}^a$ being the generators of the $SU(2)$ Lie group. It has been shown [18] that the Yang–Mills fields can be realized in certain semiconductor materials.

First, we consider a two-dimensional system (in the x–y-plane) with four-parameter Yang–Mills gauge potentials: $\vec{\mathcal{A}}_0 = (0, 0, 0)$, $\vec{\mathcal{A}}_x = \frac{2m}{\eta\hbar}(0, \beta + \alpha, \gamma_2)$, $\vec{\mathcal{A}}_y = \frac{2m}{\eta\hbar}(\beta - \alpha, 0, \gamma_1)$, $\vec{\mathcal{A}}_z = (0, 0, 0)$ where α, β and the γ s characterize the strengths of spin–orbit couplings of Rashba-type, Dresselhaus-type, etc, respectively. The couplings characterized by the γ s are relevant to the effect caused by strain exerting on inversion asymmetric semiconductor materials [19]. Writing out the Hamiltonian explicitly, we have

$$\begin{aligned} H &= \frac{\hbar^2k^2}{2m} + V + k_y\sigma_x(\alpha - \beta) - k_x\sigma_y(\alpha + \beta) - (\gamma_1 k_y + \gamma_2 k_x)\sigma_z \\ &= \frac{\hbar^2k^2}{2m} + V + \frac{\hbar}{2}\vec{\sigma}\cdot\mathbf{\Omega}_\mathbf{k}, \end{aligned} \tag{2}$$

with $V = 2m(\gamma_1^2 + 2(\alpha^2 + \beta^2) + \gamma_2^2)/\hbar^2 + V_{\rm sc}$, $\mathbf{\Omega}_\mathbf{k} = 2(k_y(\alpha - \beta), -k_x(\alpha + \beta), -(\gamma_1 k_y + \gamma_2 k_x))/\hbar$ and $\vec{\sigma}$ being the Pauli matrix. Here m stands for the effective mass of electrons in the material, and $V_{\rm sc} = eA_0$ the scattering potential, which is independent of spin indices. The scattering is supposed to be elastic. The last term in the above equation, $H' = \frac{\hbar}{2}\vec{\sigma}\cdot\mathbf{\Omega}_\mathbf{k}$, causes the precession of electron spins with the Larmor frequency $\mathbf{\Omega}_\mathbf{k}$, which can be regarded as an effective magnetic field.

We apply the density-matrix formulation proposed in [14, 19] to calculate the spin-relaxation time. The electron density matrix $\rho(\mathbf{k})$ with components $\rho_{ss'}(\mathbf{k})$, s, s' being the indices of electron spin states, is defined by

$$\frac{\rho(\mathbf{k})}{\tau} + \frac{\mathrm{i}}{\hbar}[H'(\mathbf{k}), \rho(\mathbf{k})] + \sum_{k'} W_{\mathbf{k}\mathbf{k}'}(\rho(\mathbf{k}) - \rho(\mathbf{k}')) = 0, \tag{3}$$

where τ is the life time, $W_{\mathbf{k}\mathbf{k}'}$ is the scattering probability from $\mathbf{k}$ to $\mathbf{k}'$ and the square bracket denotes commutator.

Since H' contributes merely a small perturbation, the spin-relaxation time is much longer than the time for the electron-momentum distribution to become isotropic, i.e., $\tau \gg \tau_1$, τ_1 being the momentum relaxation time. Therefore, it is convenient to split the density matrix into two parts,

$$\rho = \overline{\rho} + \rho', \qquad \text{with } \overline{\rho'} = 0. \tag{4}$$

Here we use a bar to denote an average taken over all directions of $\mathbf{k}$ and a prime to denote the deviation part, i.e., $\overline{\rho}$ depends on $\varepsilon = \hbar^2|\mathbf{k}|^2/2$ and $\rho'(\mathbf{k}) \ll \overline{\rho}$. Taking the average for equation (3), we have the following relation:

$$\frac{\overline{\rho}}{\tau} + \frac{\mathrm{i}}{\hbar}\overline{[H'(\mathbf{k}), \rho'(\mathbf{k})]} = 0. \tag{5}$$

Equation (3) can also be written out as

$$\frac{\rho'(\mathbf{k})}{\tau} + \frac{\mathrm{i}}{\hbar}[H'(\mathbf{k}), \rho'(\mathbf{k})] - \frac{\mathrm{i}}{\hbar}\overline{[H'(\mathbf{k}), \rho'(\mathbf{k})]} + \frac{\mathrm{i}}{\hbar}[H'(\mathbf{k}), \overline{\rho}] + \sum_{\mathbf{k}'} W_{\mathbf{k}\mathbf{k}'}[\rho'(\mathbf{k}) - \rho'(\mathbf{k}')] = 0, \tag{6}$$

in which equation (5) has been used. Without taking account of the higher-order terms, one needs to solve the following equation:

$$\frac{\mathrm{i}}{\hbar}[H'(\mathbf{k}), \overline{\rho}] + \sum_{\mathbf{k}'} W_{\mathbf{k}\mathbf{k}'}[\rho'(\mathbf{k}) - \rho'(\mathbf{k}')] = 0. \tag{7}$$

This approximation is valid when $\Omega_k \tau_1 \ll 1$. For an elastic process the scattering probability is a function of deflection angle only, which makes it possible to expand the above equation in terms of Fourier series. After some algebra, one can express ρ' in terms of $\overline{\rho}$. Substituting it into equation (5) and employing the Boltzmann equation with only the collision term, one can obtain

$$\left(\frac{\partial \overline{\rho}}{\partial t}\right)_{\text{sp.rel.}} = -\frac{1}{\hbar^2}\sum_n \tau_n [H'_{-n}, [H'_n, \overline{\rho}]], \tag{8}$$

with

$$H'_n = \oint \frac{\mathrm{d}\phi_{\mathbf{k}}}{2\pi} H'(\mathbf{k}) \exp(-\mathrm{i}n\phi_{\mathbf{k}}), \tag{9}$$

$$\frac{1}{\tau_n} = \oint \mathrm{d}\theta\, W_{\mathbf{k}\mathbf{k}'}(1 - \cos n\theta), \tag{10}$$

where $\phi_{\mathbf{k}}$ is the angle between $\mathbf{k}$ and the x-axis, and $\theta = \phi_{\mathbf{k}} - \phi_{\mathbf{k}'}$.

Now we are in the position to investigate the kinetics of the spin density $S_i(t) = \int a_i(\varepsilon, t)\, \mathrm{d}\varepsilon$ where $a_i = \mathrm{tr}(\bar{\rho}\sigma_i)$. Equation (8) gives rise to a differential equation for $a_i(\varepsilon, t)$. Following the factorization technique [14], $a_i(\varepsilon, t) = [F_+(\varepsilon) - F_-(\varepsilon)]s_i(t)$, where $\mathbf{s} = (s_1, s_2, s_3)$ denotes the unit vector along the spin and $F_\pm(\varepsilon)$ refer to the distribution function projected along the direction parallel or antiparallel to the unit vector. Accordingly, we obtain the evolution equation for the spin density at a time longer than τ_1 [15]:

$$\begin{aligned} \dot{S}_i(t) &= -\frac{1}{\tau_{ij}} S_j(t) \\ \frac{1}{\tau_{ij}} &= \frac{1}{2\hbar^2} \sum_{n=-\infty}^{+\infty} \frac{\int \mathrm{d}\varepsilon\, (F_+ - F_-)\tau_n \mathrm{tr}\{[H'_{-n}, [H'_n, \sigma_j]]\sigma_i\}}{\int \mathrm{d}\varepsilon (F_+ - F_-)}, \end{aligned} \tag{11}$$

where $i, j = x, y, z$.

For the Hamiltonian under consideration (2), we obtain the following:

$$\begin{aligned} \frac{1}{\tau_{xx}} &= \frac{\gamma_1^2 + \gamma_2^2 + (\alpha+\beta)^2}{2}\Lambda, \\ \frac{1}{\tau_{yy}} &= \frac{\gamma_1^2 + \gamma_2^2 + (\alpha-\beta)^2}{2}\Lambda, \\ \frac{1}{\tau_{zz}} &= (\alpha^2 + \beta^2)\Lambda, \\ \frac{1}{\tau_{xz}} &= \frac{1}{\tau_{zx}} = \frac{\gamma_1(\alpha-\beta)}{2}\Lambda, \\ \frac{1}{\tau_{yz}} &= \frac{1}{\tau_{zy}} = -\frac{\gamma_2(\alpha+\beta)}{2}\Lambda, \\ \frac{1}{\tau_{xy}} &= \frac{1}{\tau_{yx}} = 0, \end{aligned} \tag{12}$$

where the coefficient Λ is given by

$$\Lambda = \frac{8m}{\hbar^4} \frac{\int \mathrm{d}\varepsilon\, [F_+(\varepsilon) - F_-(\varepsilon)]\tau_1(\varepsilon)\varepsilon}{\int \mathrm{d}\varepsilon\, [F_+(\varepsilon) - F_-(\varepsilon)]}.$$

The above entities in equation (12) define the spin-relaxation tensor, $\Gamma = \mathrm{mat}(\frac{1}{\tau_{ij}})$. Diagonalizing this matrix, we obtain

$$\mathbb{T}^{-1} = \frac{\Lambda}{2} \begin{pmatrix} \frac{1}{\tau_\perp} & 0 & 0 \\ 0 & \frac{1}{\tau_{\parallel,+}} & 0 \\ 0 & 0 & \frac{1}{\tau_{\parallel,-}} \end{pmatrix}, \tag{13}$$

with

$$\begin{aligned} \frac{1}{\tau_{\parallel,\pm}} &= \frac{1}{2}\left\{\frac{1}{\tau_\perp} \pm \sqrt{(\gamma_1^2+\gamma_2^2)^2 + 8\alpha\beta(\gamma_2^2 - \gamma_1^2 + 2\alpha\beta)}\right\} \\ \frac{1}{\tau_\perp} &= \gamma_1^2 + \gamma_2^2 + 2(\alpha^2+\beta^2). \end{aligned} \tag{14}$$

Clearly, two of the diagonal elements are always positive definite and the other one $1/\tau_{\parallel,-}$ is not. The condition for a vanishing $1/\tau_{\parallel,-}$ turns out to be

$$\gamma_1^2(\alpha+\beta)^2 + \gamma_2^2(\alpha-\beta)^2 + (\alpha+\beta)^2(\alpha-\beta)^2 = 0. \tag{15}$$

The above equation gives rise to two solutions:

$$\begin{aligned} &\text{(i) } \alpha = \beta, && \gamma_1 = 0, \\ &\text{(ii) } \alpha = -\beta, && \gamma_2 = 0. \end{aligned} \tag{16}$$

Under these conditions, $1/\tau_{\parallel,-}$ are zero when the infinite spin-relaxation times emerge.

Actually, the Yang–Mills 'magnetic' field $\mathbb{B}_z = b^a \hat{\tau}^a$ can be calculated [18], namely

$$b_1 = \frac{4m^2}{\eta\hbar^2}(\beta+\alpha)\gamma_1,$$
$$b_2 = \frac{4m^2}{\eta\hbar^2}(\beta-\alpha)\gamma_2, \quad (17)$$
$$b_3 = \frac{4m^2}{\eta\hbar^2}(\alpha^2-\beta^2).$$

The condition (15) is also equivalent to

$$|\vec{b}|^2 = 0, \qquad \vec{b} = (b_1, b_2, b_3),$$

which implies that the module of the Yang–Mills 'magnetic' field vanishes. The Yang–Mills 'electric' field $\vec{\mathcal{E}}_i = \eta\vec{\mathcal{A}}_0 \times \vec{\mathcal{A}}_i$ is clearly null in the absence of an external magnetic field, $\mathcal{A}_0 = 0$. Thus we see that the spin-relaxation time can be infinitely large when the Yang–Mills fields are zero. To observe the role that the terms neglected in equation (6) may play, we take the higher-order terms into account by means of an iteration method. We find the third-order contribution to the spin-relaxation tensor is zero and then calculate the fourth order. Our result is given in appendix B. One can see that the fourth-order contribution at the vicinity that conditions (16) fulfil is still negligible. This result is expected to be a criterion to evaluate whether there is an infinite spin-relaxation time in two-dimensional systems with spin–orbit couplings. In order to determine which spin component has an infinite life time, the spin precession needs to be analysed concretely.

The nonvanishing spin–orbit coupling γ_1 or γ_2 will bring about some new features which may be useful for possible design with an infinite spin-relaxation time. For the first case $\alpha = \beta$ and $\gamma_1 = 0$ in equation (16), the Hamiltonian H' reduces to

$$H' = \frac{\hbar}{2}\vec{\sigma}\cdot\mathbf{\Omega_k} = -k_x(2\alpha\sigma_y + \gamma_2\sigma_z). \quad (18)$$

The orientation of Larmor precession frequency $\mathbf{\Omega_k}$ is parallel to z'-axis as illustrated in figure A.1 in appendix A, thus one can understand why the life time of the $S_{z'}$ component is infinite while the other two decay. Here the z'-axis is defined in the diagonalization procedure of the spin-relaxation tensor equation (12) for the first case given in appendix A.

From appendix A and the figure therein, we can see that the strengths of spin–orbit coupling γ_2 and α determine the angle θ' between $\mathbf{\Omega_k}$ and the z-axis. The angle θ' can be manipulated by these two parameters; thus a definite alignment of spin with infinite life time can be realized with the help of tuning the spin–orbit coupling strengths. On the other hand, the ratio of the different types of spin–orbit coupling constant can be determined by means of measuring the spin-relaxation time experimentally.

For the second case in equation (16), a similar analysis can be carried out, which is omitted here. In the special case when both γ_1 and γ_2 vanish, one component of the tensor of the spin-relaxation time becomes zero, i.e., $1/\tau_{xx} = 0$ for $\alpha = -\beta$ or $1/\tau_{yy} = 0$ for $\alpha = \beta$, and thus the S_x or S_y has infinite life time, which is just the case considered in [14].

3. Spin relaxation affected by an external magnetic field

In the previous section, we considered the case of $\vec{\mathcal{A}}_0 = 0$, which means that an external magnetic field is absent. In the presence of a uniform magnetic field, we should take account of

$$\vec{\mathcal{A}}_0 = -\frac{2\mu_B}{\eta}(B_x, B_y, B_z), \quad (19)$$

where μ_B is the Bohr magneton. The existence of an external magnetic field is known to affect the dynamics of the electron's spin. The Larmor precession of an electron's spin around a sufficiently strong longitudinal magnetic field will suppress the precession about the internal random magnetic fields [20]. The cyclotron motion will change the wavevector **k** and affect the spin relaxation due to the D'yakonov–Perel mechanism. The density-matrix formalism is applicable for calculating the spin-relaxation time of electrons. Following [16], we expand the density matrix for electrons in terms of the unit and Pauli matrices,

$$\rho(\mathbf{k}) = f_{\mathbf{k}} + \mathbf{s}_{\mathbf{k}} \cdot \vec{\sigma}, \tag{20}$$

where $f_{\mathbf{k}} = \mathrm{tr}[\rho(\mathbf{k})/2]$ is the spin-averaged-electron-distribution function and $\mathbf{s}_{\mathbf{k}} = \mathrm{tr}[\rho(\mathbf{k})\vec{\sigma}/2]$ is the spin per **k**-state electron. The kinetic equation for the spin distribution is given by [16, 21, 22]:

$$\frac{\partial \mathbf{s}_{\mathbf{k}}}{\partial t} + \mathbf{s}_{\mathbf{k}} \times (\vec{\omega}_{\mathrm{L}} + \mathbf{\Omega}_{\mathbf{k}}) + \vec{\omega}_{\mathrm{C}} \cdot [\mathbf{k} \times \nabla_{\mathbf{k}} \mathbf{s}_{\mathbf{k}}] + \sum_{\mathbf{k}'} W_{\mathbf{k}\mathbf{k}'}(\mathbf{s}_{\mathbf{k}} - \mathbf{s}_{\mathbf{k}'}) = 0, \tag{21}$$

in which the second term refers to spin precession caused by spin–orbit couplings given in equation (2) together with the external magnetic field; the third term is related to the wavevector variations due to the cyclotron motion, and the last term denotes the collision integral. Here $\vec{\omega}_{\mathrm{L}}$ is the Larmor frequency vector and $\vec{\omega}_{\mathrm{C}}$ the cyclotron frequency vector along the growth axis [001] with $|\ \vec{\omega}_{\mathrm{C}}\ | = \frac{eB_z}{mc}$, and B_z the perpendicular component of the magnetic field. Since the internal random magnetic field is regarded as a perturbation, i.e., $\mathbf{\Omega}_{\mathbf{k}}\tau_1 \ll 1$, we can split the spin distribution function $\mathbf{s}_{\mathbf{k}}$ into the following,

$$\mathbf{s}_{\mathbf{k}} = \mathbf{s}_k^0 + \delta \mathbf{s}_{\mathbf{k}}, \tag{22}$$

where $\mathbf{s}_k^0$ is a quasi-equilibrium distribution function and thus is independent of the direction of **k**. In contrast, $\delta\mathbf{s}_{\mathbf{k}}$ is a nonequilibrium correction arising from spin–orbit couplings as well as other internal random magnetic fields, and thus it contains only the first angular harmonics of the spin distribution [23] because only elastic scattering processes are taken into account; accordingly

$$\delta \mathbf{s}_{\mathbf{k}} = \mathbf{R}_1 \cos(\phi_k) + \mathbf{R}_2 \sin(\phi_k), \tag{23}$$

where the two vectors $\mathbf{R}_1$ and $\mathbf{R}_2$ are irrelevant to the direction of the wavevector **k** though they are functions of the module of **k** in general. Substituting equations (22) and (23) into equation (21), we obtain the following equations:

$$\frac{\mathrm{d}\mathbf{s}_k^0}{\mathrm{d}t} + \mathbf{s}_k^0 \times \vec{\omega}_{\mathrm{L}} + \delta \mathbf{s}_{\mathbf{k}} \times \mathbf{\Omega}_{\mathbf{k}} = 0, \tag{24}$$

$$\frac{\mathrm{d}\delta \mathbf{s}_k}{\mathrm{d}t} + \mathbf{s}_k^0 \times \mathbf{\Omega}_{\mathbf{k}} + \delta \mathbf{s}_{\mathbf{k}} \times \vec{\omega}_{\mathrm{L}} + \vec{\omega}_{\mathrm{C}} \cdot [\mathbf{k} \times \nabla_{\mathbf{k}} \delta \mathbf{s}_{\mathbf{k}}] + \frac{\delta \mathbf{s}_{\mathbf{k}}}{\tau_1} = 0, \tag{25}$$

where τ_1 is the momentum-relaxation time whose definition is also given by equation (10) for $n = 1$. In the light of the number of total electrons $N = 2\sum_{\mathbf{k}} f_k$ and the single electron spin $\mathbf{S}^0 = \frac{\sum_{\mathbf{k}} \mathbf{s}_{\mathbf{k}}}{N}$, summing equation (24) over the wavevectors, we obtain the balance equation describing the electron's spin relaxation:

$$\frac{\mathrm{d}\mathbf{S}^0}{\mathrm{d}t} + \mathbf{S}^0 \times \vec{\omega}_{\mathrm{L}} + \hat{\Gamma}\mathbf{S}^0 = 0, \tag{26}$$

where the spin-relaxation tensor $\hat{\Gamma}$ referring to the inverse of the spin-relaxation times is defined as

$$\hat{\Gamma}\mathbf{S}^0 = \frac{1}{N} \sum_{\mathbf{k}} \delta \mathbf{s}_k \times \mathbf{\Omega}_{\mathbf{k}}. \tag{27}$$

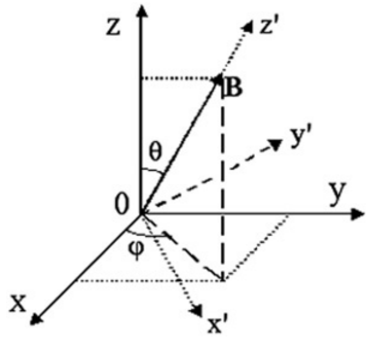

Figure 1. The scheme of coordinate frames. z-axis parallel to the [001] growth axis. θ and φ are the polar and the azimuthal angles of the external magnetic field **B**. z' is chosen in alignment with the orientation of **B**, y' lies in the x–y-plane, and x' is chosen to form a right-hand triple with y' and z'.

The nonequilibrium correction $\delta\mathbf{s_k}$ can be obtained from equation (25) in which the contribution of the rate $\mathrm{d}\delta\mathbf{s_k}/\mathrm{d}t$ is negligible because its magnitude is of higher order in $\mathbf{\Omega_k}\tau_1$.

First, we rotate the original coordinate system $\{\hat{x}, \hat{y}, \hat{z}\}$, which is related to the principal crystal axes to the new one $\{\hat{x}', \hat{y}', \hat{z}'\}$ (illustrated in figure 1). The coordinates in both systems are related, $(\hat{x}, \hat{y}, \hat{z}) = (\hat{x}', \hat{y}', \hat{z}')R^{\mathrm{T}}$, by

$$R = \begin{pmatrix} \cos\theta\cos\varphi, & -\sin\varphi, & \sin\theta\cos\varphi \\ \cos\theta\sin\varphi, & \cos\varphi, & \sin\theta\sin\varphi \\ -\sin\theta, & 0, & \cos\theta \end{pmatrix}. \tag{28}$$

Here, θ is the angle between z and z', and φ the angle between y' and y ([010]). Similar equations are valid for the momentum components, $k_i = R_{ij}k'_j$ (here $i, j = x, y, z$). It is convenient to calculate the nonequilibrium correction $\delta\mathbf{s_k}$ and the components of the spin-relaxation tensor in the new frame of coordinates where the Larmor frequency vector $\vec{\omega}_{\mathrm{L}} = \frac{\mu_{\mathrm{B}}}{\hbar}B_j\hat{e}_j$ in the original coordinates becomes $\vec{\omega}_{\mathrm{L}} = \omega_{\mathrm{L}}\hat{z}'$ in the new coordinates.

After tedious calculation, we obtain the spin-relaxation tensor (inverse of the spin-relaxation time) $\hat{\mathbf{\Gamma}}$ for degenerate electrons with Fermi energy E_{F}, which is given in appendix C. These results are valid for arbitrary random internal magnetic field and arbitrary orientation of the external field, from which we obtain several conclusions that will be illustrated in turn.

3.1. Longitudinal relaxation

The longitudinal spin-relaxation rate is $1/\tau_{\mathrm{L}} = \Gamma_{z'z'}$. From equation (C.1), we can obtain the following conclusion: $\Gamma_{z'z'} = 0$ when either

$$\alpha = \beta, \quad \gamma_1 = 0, \quad \varphi = \pi/2, \quad \theta = \tan^{-1}(2\alpha/\gamma_2), \tag{29a}$$

or

$$\alpha = -\beta, \quad \gamma_2 = 0, \quad \varphi = 0, \quad \theta = -\tan^{-1}(2\alpha/\gamma_1). \tag{29b}$$

When the longitudinal spin-relaxation time τ_{L} is infinite, the spin component $S_{z'}$ has an infinite life time. Certainly, the Hamiltonian H' describing the electron spin precession arising from spin–orbit coupling can also be written as equation (18) for the case in equation (29a). One can see from figure A.1 that $S_{z'}$ is the component parallel to $\mathbf{\Omega_k}$ when $\theta' = \theta = \tan^{-1}(2\alpha/\gamma_2)$ and $\varphi = \pi/2$. Thus the infinite life time of $S_{z'}$ can easily be understood from a physical point of view; that is to say, $S_{z'}$ will not precess about the Larmor precession frequency $\mathbf{\Omega_k}$ and the external magnetic field **B** when $\mathbf{\Omega_k}$ is parallel to **B**. However, the other components, $S_{x'}$ and $S_{y'}$, have finite life time due to the precession around $\mathbf{\Omega_k}$ arising from the internal random magnetic field. If the strength of both spin–orbit couplings γ_1 and γ_2 vanishes, the longitudinal component of the tensor $\Gamma_{z'z'}$ is zero when $\theta = \pi/2$, $\varphi = \pi/2$, which can be seen from equation (C.1). This recovers the special case discussed in [16].

For the latter case in equation (29*b*), the Hamiltonian H' becomes

$$H' = \frac{\hbar}{2}\vec{\sigma} \cdot \mathbf{\Omega_k} = k_y(2\alpha\sigma_x - \gamma_1\sigma_z). \tag{30}$$

The Larmor frequency $\mathbf{\Omega_k} = \frac{2k_y}{\hbar}(2\alpha,\ 0,\ -\gamma_1)$ is parallel to the external magnetic field $\mathbf{B}(\theta = -\tan^{-1}(2\alpha/\gamma_1), \varphi = 0)$. So $S_{z'}$ does not decay because $\mathbf{\Omega_k}$ is parallel to **B**.

We can verify from the relations given by equations (29*b*) that both the Yang–Mills ‘magnetic’ and ‘electric’ fields are zero. Note that the restrictions on the θ and φ related to external magnetic field are precisely the same condition for the Yang–Mills ‘electric’ field being zero. Therefore, vanishing Yang–Mills fields can still be a criterion for the existence of infinite spin-relaxation time in the presence of an external magnetic field.

3.2. Transverse relaxation

Let us analyse the spin relaxation in the plane perpendicular to the external magnetic field. One can find that the transverse components of the spin-relaxation tensor can also be zero (i.e., $\Gamma_{x'x'} = 0$, $\Gamma_{x'y'} = 0$, $\Gamma_{y'x'} = 0$) when either

$$\alpha = \beta, \quad \gamma_1 = 0, \quad \varphi = \pi/2, \theta = -\tan^{-1}\left(\frac{\gamma_2}{2\alpha}\right), \tag{31a}$$

or

$$\alpha = -\beta, \quad \gamma_2 = 0, \quad \varphi = 0, \theta = \tan^{-1}\left(\frac{\gamma_1}{2\alpha}\right). \tag{31b}$$

Here the Yang–Mills ‘magnetic’ field is zero but its ‘electric’ field is not zero; hence infinite spin-relaxation time does not occur for $\Gamma_{z'z'} \neq 0$.

As shown in figure 2, the x'-axis is antiparallel to the Larmor frequency (i.e., $\hat{x}' \parallel -\mathbf{\Omega_k}$) for the case in equation (31*a*). The spin component $S_{x'}$ does not precess about the Larmor precession frequency $\mathbf{\Omega_k}$. We know that **B** is perpendicular to $\mathbf{\Omega_k}$ from figure 2; thus $S_{x'}$ will precess about the constant external magnetic field **B** in the plane parallel to $\mathbf{\Omega_k}$. So the random internal magnetic fields and external magnetic field cannot induce spin relaxation for the spin component $S_{x'}$ due to $\Gamma_{x'x'} = 0$. And the admixture of the x' component to the y' component of spin-relaxation tensor (which is described by $\Gamma_{x'y'}$, $\Gamma_{y'x'}$) is zero, namely $\Gamma_{x'y'} = 0$ and $\Gamma_{y'x'} = 0$, which can also be calculated from equations (C.1)–(C.5).

For the latter case in equation (31*b*), the direction of the external magnetic field is also perpendicular to the Larmor frequency $\mathbf{\Omega_k}$, as illustrated in equation (30). The spin component $S_{x'}$ is antiparallel to the Larmor frequency ($\hat{x}' \parallel -\mathbf{\Omega_k}$). The random internal magnetic field and external magnetic field will not induce spin relaxation for the spin component $S_{x'}$ for the same reason as mentioned before.

The components $\Gamma_{x'z'}$ and $\Gamma_{y'z'}$ are smaller than others when the external magnetic field is sufficiently strong ($\Omega_k^2\tau_1 \ll \boldsymbol{\omega}_\mathbf{L}$). Under this condition the in-plane spin components rapidly rotate and the admixture of the in-plane components to z'-component (which is described by $\Gamma_{x'z'}$, $\Gamma_{y'z'}$) plays no role in the spin dynamics. Therefore the above result manifests the general solutions of spin-relaxation time for the Hamiltonian equation (2).

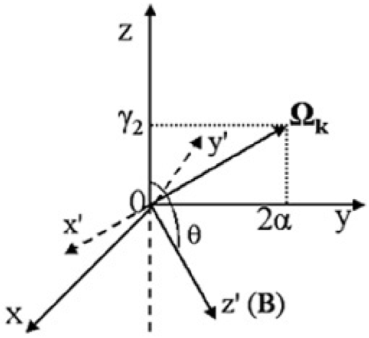

Figure 2. Schematic of the case $\Gamma_{x'x'} = 0$. $\alpha = \beta$, $\gamma_1 = 0$, $\varphi = \pi/2$, $\theta = \tan^{-1}(-\gamma_2/2\alpha)$; the external magnetic field **B** is parallel to the z'-direction, and the x'-axis is antiparallel to the Larmor frequency $\mathbf{\Omega_k}$.

4. Summary

In the above, we have developed a consistent theory of spin dynamics to describe particles moving in an external $U(1)$ Maxwell field and $SU(2)$ Yang–Mills fields which characterizes spin–orbit couplings in certain semiconductors (such as Rashba-type, Dresselhaus-type, strain-induced or other complex types). We used the density-matrix formalism to calculate the spin-relaxation time in such systems in the absence and in the presence of an external magnetic field, respectively. In the absence of an external magnetic field, we find that the spin component $S_{z'}$ has an infinite life time if the strengths of spin–orbit couplings α, β, γ_1 and γ_2 satisfy either (i) $\alpha = \beta$, $\gamma_1 = 0$ or (ii) $\alpha = -\beta$, $\gamma_2 = 0$. In such a case, the Yang–Mills fields vanish. From these conditions, the direction of the spin component with infinite life time can be manipulated by tuning α, γ_2 or β, γ_1 respectively. In the presence of an external magnetic field, we considered the magnetic effect on a two-dimensional system. We obtained that the longitudinal spin-relaxation time is infinite when $S_{z'}$ is parallel to $\mathbf{\Omega_k}$ and **B** if either (i′) $\alpha = \beta$, $\gamma_1 = 0$, $\varphi = \pi/2$, $\theta = \tan^{-1}(2\alpha/\gamma_2)$ or (ii′) $\alpha = -\beta$, $\gamma_2 = 0$, $\varphi = 0$, $\theta = -\tan^{-1}(2\alpha/\gamma_1)$. We noticed that the vanishing Yang–Mills fields can be a criterion for the existence of infinite spin-relaxation time. We also analysed the spin relaxation for the in-plane spin component $S_{x'}$. The external magnetic field cannot induce spin relaxation for $S_{x'}$ if it is antiparallel to $\mathbf{\Omega_k}$ and perpendicular to **B** for either (iii′) $\alpha = \beta$, $\gamma_1 = 0$, $\varphi = \pi/2$, $\theta = \tan^{-1}(-\gamma_2/2\alpha)$ or (iv′) $\alpha = -\beta$, $\gamma_2 = 0$, $\varphi = 0$, $\theta = \tan^{-1}(\gamma_1/2\alpha)$. These solutions provide a better understanding on the spin dynamics of a two-dimensional system with four-parameter Yang–Mills potentials, which characterizes a hierarchy of spin–orbit coupling in certain semiconductor materials. It is expected to expose some more clues for manipulating spin via certain spin–orbit couplings in semiconductors or elaborating spintronics storage devices with long spin-relaxation time. Note that other mechanisms will be necessary for systems that cannot be characterized by the aforementioned Yang–Mills field formalism.

Acknowledgments

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Appendix A. On the diagonalizing bases

The spin-relaxation tensor given by equation (12) is diagonalized to be equation (13) by a U matrix,

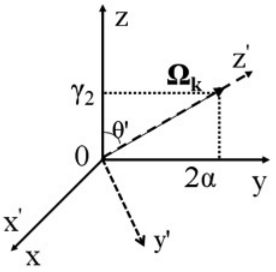

Figure A.1. The orientation of the Larmor frequency $\boldsymbol{\Omega}_{\mathbf{k}}$ for the case $\alpha = \beta$, $\gamma_1 = 0$, which defines the z'-axis of the new frame of coordinates.

$$\mathbb{T}^{-1} = U^{-1}\Gamma\, U. \tag{A.1}$$

The matrix U turns the evolution equation for the spin density, equation (11), to be

$$\begin{aligned}\frac{\mathrm{d}S'}{\mathrm{d}t} &= U^{-1}\frac{\mathrm{d}S}{\mathrm{d}t} = -(U^{-1}\Gamma\, U)U^{-1}S\\ &= -\mathbb{T}^{-1}S'.\end{aligned} \tag{A.2}$$

Hence, the new spin components $S' = U^{-1}S$ can be obtained. With particular interest for $\alpha = \beta$, $\gamma_1 = 0$, we write out the U matrix:

$$U = \begin{pmatrix} 1 & 0 & 0\\ 0 & \frac{\gamma_2}{\sqrt{\gamma_2^2+4\alpha^2}} & \frac{2\alpha}{\sqrt{\gamma_2^2+4\alpha^2}}\\ 0 & \frac{-2\alpha}{\sqrt{\gamma_2^2+4\alpha^2}} & \frac{\gamma_2}{\sqrt{\gamma_2^2+4\alpha^2}}\end{pmatrix}; \tag{A.3}$$

then we have

$$\begin{aligned}S'_x &= S_x,\\ S'_y &= \cos\theta' S_y - \sin\theta' S_z,\\ S'_z &= \sin\theta' S_y + \cos\theta' S_z,\end{aligned}$$

where $\tan\theta' = 2\alpha/\gamma_2$. This means the existence of spin–orbit coupling γ_2 makes the orientation of the Larmor frequency $\boldsymbol{\Omega}_{\mathbf{k}}$ to change from the y-axis to the z'-axis. As illustrated in figure A.1, θ' refers to the angle between the Larmor frequency and the z-axis.

Appendix B. Higher-order contributions

The evolution equation for the spin density up to fourth order reads

$$\begin{aligned}\dot{S}_i(t) &= -\frac{1}{\tau'_{ij}}S_j(t)\\ \frac{1}{\tau'_{ij}} &= \frac{1}{2\hbar^2}\sum_{n=-\infty}^{+\infty}\frac{\int \mathrm{d}\varepsilon\,(F_+ - F_-)\tau_n \mathrm{tr}\{[H'_{-n},\,[H'_n,\,\sigma_j]\,]\sigma_i\}}{\int \mathrm{d}\varepsilon\,(F_+ - F_-)} - \frac{1}{2\hbar^4}\sum_{n=-\infty}^{+\infty}\\ &\quad\times\frac{\int \mathrm{d}\varepsilon(F_+ - F_-)\tau_n \mathrm{tr}\{[H'_{-n},\,\sum_{n''}[H'_{n-n''},\,\tau_{n''}\sum_{n'}[H'_{n''-n'},\,\tau_{n'}[H'_{n'},\,\sigma_j]\,]\,]\,]\sigma_i\}}{\int \mathrm{d}\varepsilon(F_+ - F_-)}\end{aligned} \tag{B.1}$$

where $i, j = x, y, z$. For the Hamiltonian (2) we obtain

$$
\begin{aligned}
\frac{1}{\tau'_{xx}} &= \frac{1}{\tau_{xx}} - [(\alpha-\beta)^2((\alpha+\beta)^2 + (\gamma_1^2+\gamma_2^2)) + ((\alpha+\beta)^2 - \gamma_1^2 + \gamma_2^2)^2 + 4\gamma_1^2\gamma_2^2]\Lambda', \\
\frac{1}{\tau'_{yy}} &= \frac{1}{\tau_{yy}} - [(\alpha-\beta)^4 + (\alpha-\beta)^2((\alpha+\beta)^2 + 2(\gamma_1^2-\gamma_2^2)) \\
&\quad + (\alpha+\beta)^2(\gamma_1^2+\gamma_2^2) + (\gamma_1^2+\gamma_2^2)^2]\Lambda', \\
\frac{1}{\tau'_{zz}} &= \frac{1}{\tau_{zz}} - [(\alpha+\beta)^4 + (\alpha-\beta)^4 + (\alpha+\beta)^2(\gamma_1^2+\gamma_2^2) \\
&\quad + (\alpha-\beta)^2(\gamma_1^2+\gamma_2^2 - 2(\alpha+\beta)^2)]\Lambda', \\
\frac{1}{\tau'_{xz}} &= \frac{1}{\tau'_{zx}} = \frac{1}{\tau_{xz}} - [(\alpha-\beta)^2 - 3(\alpha+\beta)^2 + \gamma_1^2 + \gamma_2^2]\gamma_1(\alpha-\beta)\Lambda', \\
\frac{1}{\tau'_{yz}} &= \frac{1}{\tau'_{zy}} = \frac{1}{\tau_{yz}} + [(\alpha+\beta)^2 - 3(\alpha-\beta)^2 + \gamma_1^2 + \gamma_2^2]\gamma_2(\alpha+\beta)\Lambda', \\
\frac{1}{\tau'_{xy}} &= \frac{1}{\tau'_{yx}} = \frac{1}{\tau_{xy}} - 4\gamma_1\gamma_2(\alpha-\beta)(\alpha+\beta)\Lambda',
\end{aligned}
\tag{B.2}
$$

with $1/\tau_{ij}$ the same as in equation (12) and Λ' given by

$$
\Lambda' = \frac{8m^2}{\hbar^8}\frac{\int \mathrm{d}\varepsilon\,[F_+(\varepsilon) - F_-(\varepsilon)]\tau_1^2(\varepsilon)\tau_2(\varepsilon)\varepsilon^2}{\int \mathrm{d}\varepsilon\,[F_+(\varepsilon) - F_-(\varepsilon)]}.
$$

Appendix C. The spin-relaxation tensor with magnetic field

In the presence of a magnetic field, we obtained the following spin-relaxation tensor:

$$
\begin{aligned}
\Gamma_{z'z'} &= \frac{2k^2\tau_1}{\hbar^2 D_+D_-}\{[1+\tau_1^2(\omega_C^2+\omega_L^2)][(\alpha+\beta)^2\cos^2\varphi + (\alpha-\beta)^2\sin^2\varphi \\
&\quad + (\gamma_2\sin\theta - (\alpha+\beta)\cos\theta\sin\varphi)^2 + (\gamma_1\sin\theta + (\alpha-\beta)\cos\theta\cos\varphi)^2] \\
&\quad + 4\tau_1^2\omega_C\omega_L[(\alpha^2-\beta^2)\cos\theta + (\alpha+\beta)\gamma_1\sin\theta\cos\varphi \\
&\quad + (\beta-\alpha)\gamma_2\sin\theta\sin\varphi]\}
\end{aligned}
\tag{C.1}
$$

$$
\begin{aligned}
\Gamma_{x'x'} &= \frac{2k^2\tau_1}{\hbar^2}\left\{\frac{1+(\omega_C^2+\omega_L^2)\tau_1^2}{D_+D_-}[(\gamma_2\cos\theta + (\alpha+\beta)\sin\theta\sin\varphi)^2\right. \\
&\quad \left. + (\gamma_1\cos\theta - (\alpha-\beta)\sin\theta\cos\varphi)^2] + \frac{(\alpha-\beta)^2\sin^2\varphi + (\alpha+\beta)^2\cos^2\varphi}{1+\omega_C^2\tau_1^2}\right\}
\end{aligned}
\tag{C.2}
$$

$$
\begin{aligned}
\Gamma_{y'y'} &= \frac{2k^2\tau_1}{\hbar^2}\left\{\frac{1+(\omega_C^2+\omega_L^2)\tau_1^2}{D_+D_-}[(\gamma_2\cos\theta + (\alpha+\beta)\sin\theta\sin\varphi)^2\right. \\
&\quad + (\gamma_1\cos\theta - (\alpha-\beta)\sin\theta\cos\varphi)^2] \\
&\quad + \frac{1}{1+\omega_C^2\tau_1^2}[(\gamma_2\sin\theta - (\alpha+\beta)\cos\theta\sin\varphi)^2 \\
&\quad \left. + (\gamma_1\sin\theta + (\alpha-\beta)\cos\theta\cos\varphi)^2]\right\}
\end{aligned}
\tag{C.3}
$$

$$
\Gamma_{x'y'} = \frac{2k^2\tau_1}{\hbar^2}\left\{\frac{[(\omega_C^2-\omega_L^2)\tau_1^2 - 1]\omega_L\tau_1}{D_+D_-}[(\gamma_2\cos\theta + (\alpha+\beta)\sin\theta\sin\varphi)^2\right.
$$

$$
\begin{aligned}
&+ (\gamma_1 \cos\theta - (\alpha-\beta)\sin\theta\cos\varphi)^2] \\
&+ \frac{1}{1+\omega_C^2\tau_1^2}[-(\alpha^2-\beta^2)\cos\theta\omega_C\tau_1 + ((\alpha-\beta)^2 - (\alpha+\beta)^2)\cos\theta\cos\varphi\sin\varphi \\
&+ \gamma_1[(\alpha-\beta)\sin\theta\sin\varphi - (\alpha+\beta)\omega_C\tau_1\sin\theta\cos\varphi] \\
&+ \gamma_2[(\alpha-\beta)\omega_C\tau_1\sin\theta\sin\varphi + (\alpha+\beta)\sin\theta\cos\varphi]]\Big\}
\end{aligned}
\tag{C.4}
$$

$$
\begin{aligned}
\Gamma_{y'x'} = \frac{2k^2\tau_1}{\hbar^2}\Big\{&\frac{[1-(\omega_C^2-\omega_L^2)\tau_1^2]\omega_L\tau_1}{D_+D_-}[(\gamma_2\cos\theta + (\alpha+\beta)\sin\theta\sin\varphi)^2 \\
&+ (\gamma_1 \cos\theta - (\alpha-\beta)\sin\theta\cos\varphi)^2] \\
&+ \frac{1}{1+\omega_C^2\tau_1^2}[(\alpha^2-\beta^2)\cos\theta\omega_C\tau_1 + ((\alpha-\beta)^2 - (\alpha+\beta)^2)\cos\theta\cos\varphi\sin\varphi \\
&+ \gamma_1[(\alpha-\beta)\sin\theta\sin\varphi + (\alpha+\beta)\omega_C\tau_1\sin\theta\cos\varphi] \\
&+ \gamma_2[(\alpha-\beta)\omega_C\tau_1\sin\theta\sin\varphi + (\alpha+\beta)\sin\theta\cos\varphi]]\Big\}
\end{aligned}
\tag{C.5}
$$

with $k = \sqrt{2mE_F}$ and $D_\pm = 1 + (\omega_L \pm \omega_C)^2\tau_1^2$.

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